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Active Prototyping: 
A Computational Framework for Designing while Making

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Abstract

In the wake of an increased accessibility of rapid prototyping tools in design education and practice, designers still face a series of challenges related to their use, one of them being the way in which they use these machines to actively explore and enhance their ideas. At the same time, the concepts of continuous interaction with computational fabrication tools and design exploration through physical prototyping are gaining impetus in computational design research and human-computer interaction. Stimulated by these inquiries, the hypothesis of this thesis is that physical prototyping tools can be used as tools for active design exploration and evaluation. Towards this goal, I introduce Active Prototyping, a framework for enhancing physical engagement with design objects by aiding the designer to project the impact of tools on design outcomes and explore a range of possible design solutions while making.

Active Prototyping integrates the following operations: (a) physical control of a fabrication device, (b) recording of designer actions while using the device (c) visual exploration of possible design solutions while developing a physical prototype and (d) machine feedback on the prototyping of selected design solutions. To demonstrate the Active Prototyping framework, I develop Fabcorder, a technical apparatus that implements a number of the above operations. Through application examples, I demonstrate how Active Prototyping can render physical prototyping processes more exploratory and digital fabrication processes more intuitive. I conclude by proposing action recording and generative methods as two novel additions to existing frameworks for computational design and fabrication that can bring future tool-making strategies into a more creative context.

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Appendix
1. Introduction
1.1 Hypothesis

Since their early introduction during the 1950s, machines for rapid prototyping, for example 3d-printers and CNC machines have been used to facilitate the fabrication process across different design disciplines, such as architecture, mechanical engineering and industrial design. In recent years, the vision of Personal Fabrication (Gershenfeld 2015) coalesced with the expansion of the Maker Movement (Hatch 2014), as well as important changes in the licensing of previously patented tools (Baudisch and Mueller 2017), yielding the dissemination of machines for rapid prototyping beyond the boundaries of the design disciplines.

This ubiquity of rapid prototyping tools increased the use of machines for rapid prototyping within the design disciplines, too. However, in the wake of this increased accessibility, designers who use machines for rapid prototyping in their everyday practice still face high production and maintenance costs, results that are often of uncertain quality—as they highly depend on the capabilities and the calibration of machines—, and, most critically, lack of physical interaction while the prototype is being produced. For example, in an architecture studio, the printing of a small-scale prototype can take up to twenty hours to be completed. Typically, the time spent during the prototyping process is inactive on the part of the designer, as a prototype cannot be re-designed, changed and improved until the 3d-printing process is complete. If the designer decides to make design changes, the fabrication process has to start from scratch and be repeated until the result satisfies the designer goals. Moreover, there is lack of compatibility with regards to physical design inputs. If the designer wants to experiment on a physical prototype or a sketch before creating a digital model, she has to find ways to digitize it first. This back and forth process between designing and making represents not only a delay in realizing a design outcome, but also one in being able to reflect on it and evaluate it while making, an activity which is integral to design as discussed in relevant literature (Schön 1983; Schön 1985; Anthony 1991; Smith 2000). In a broader sense, it lies in the fact that, in a computer-aided prototyping process, making is detached from design.
Even anticipating technical advances that will reduce fabrication time and costs, the problem of interaction with design prototypes, and in particular the problem of exploring design while making using machines for rapid prototyping, remains a conceptual and technical challenge. Towards the solution of this problem, I introduce Active Prototyping, a computational framework in which prototyping tools are used as tools for design exploration and evaluation. The idea motivating the Active Prototyping framework is that, by collecting information related to the use of tools during the development of a physical prototype, designers can archive the design knowledge that these actions encompass, and by further processing this information using generative design systems, they can explore, control and enhance design outcomes. I introduce Active Prototyping, a computational framework that enables designers to:

- Physically engage with design objects during the process of prototyping.

- Record their actions and their translations into design outcomes.

- Explore a range of design ideas and choose among them.

- Get feedback and assistance on the physical implementation of designs.

Active prototyping is introduced at a time and in a technological reality where the way that designers learn and improve their artifacts through in-situ, real-time prototyping practices (tool-using) is being widely investigated. At the same time, it is inspired by new systems for integrating prototyping machines into the design process (tool-making) that are mostly developed within the fields of engineering and human-computer interaction. In particular,

- In prototyping research, studies mainly focus on tool-using practices. The use of physical mediums, such as sketches, gestures, material units and actions in combination with computational design and making tools is a common direction among current inquiries. Approaches vary across different design disciplines.
In architectural design and design theory, they range from conceptual (McCullough 1998) to formal (Knight and Stiny 2015) to technical (Johns et al. 2014), expressing different attitudes and levels of enthusiasm with technological tools. In industrial design and in engineering, research examines the efficiency of the design outcome as a result of the material and crafting procedures followed by the designer: interaction with physical prototypes is viewed as a learning tool (Yang 2005), or as a medium for evaluation and selection of possible solutions (Ward et. al 1995). Among these approaches, focus is given on how findings can be used to enhance current making and prototyping methods, rather than to create new tools.

- At the same time, systems for integrating the same mediums (sketches, gestures, etc.) in the process of prototyping have recently been developed in the field of human-computer interaction. This branch of research, referred to as interactive fabrication (Willis et al. 2011) views tools for fabrication as embodied interfaces that can make design, prototyping and fabrication accessible to non-designers, as well as more intuitive and flexible for designers. The process of prototyping in these systems largely depends on the capabilities of the tools, so focus is given to the technical specifications that make the tool operations possible. In particular, current interactive fabrication systems focus on the facilitation of the fabrication processes through the translation of physical mediums into computer inputs and outputs (Baudisch and Mueller 2017; Peng et al. 2016; Peng et al 2018), or on the introduction of traditionally digital functions, such as reversing (“undoing”) operations (Weichel et al. 2015). User testing as well as documented demonstrations of interactive fabrication tools have shown what tools can do more efficiently than the hands of a novice user, however more is to be explored on how tools can enhance the practices of experienced users or learn from them.
A comparison between current tool-using and tool-making approaches indicates that analysis of what makes a prototyping procedure creative and invention of new techniques for efficient control of prototyping tools are deployed as two remote enterprises. Based on an analysis of current divergences and convergences between these two directions, I propose Active Prototyping as a framework for integrating tool-making agendas with tool-using practices. This objective stems from the observation that tools for making, both physical and computational, and the associated ways in which these tools affect the user’s creative process and operation lie in the intersection of this distance: on the one hand there is a rich variety of findings on tool using, and on the other there is an emerging expansion on tool-making. How then can we use knowledge deriving from prototyping activity (tool-using) to build functional tools engaging design creativity and thinking (tool-making)? Moreover, how can we take advantage of the computational features of storage, automation and feedback towards informing and enhancing our physical actions when making?

These questions call upon the integration of systematic methods for design exploration into the workflow of prototyping tools. In my proposal, I introduce these methods through generative design frameworks. While research on prototyping tools and activity demonstrates how design can be enhanced through physical making, frameworks for generative design propose a computational, representation-based way for enhancing design. The question that then rises is: how can prototyping tools be used
in a generative way? Active Prototyping takes advantage of the main conceptual and technical tools of generative design methods such as rule applications, solution enumeration and selection in order to answer this question.

Stimulated by these inquiries, the hypothesis of this thesis is that prototyping tools can be used as tools for active design exploration and evaluation through the integration of generative methods into their workflows. If we want to develop new ways of extracting information from our practice and turn it into applicable knowledge, we need to consider the different kinds of mechanisms - conceptual, technical, formal and empirical - that will allow this transformation to reach its full potential. Towards this goal, the hypothesis of this thesis will be supported by (i) an up-to-date analysis of current findings in design research and personal fabrication, (ii) the formulation the Active Prototyping framework as a novel scheme for enhancing the use of physical prototyping tools through methods for digital design exploration and (iii) the development of Fabcorder, a human-machine interface that implements this framework through a number of technical procedures such as motion tracking technologies, computational search of possible design outcomes, and interfaces for visual output.

1.2 Method

The proposal of developing a human-machine prototyping framework based on research coming from these diverse backgrounds does not reflect an intention for creating a hybrid assembly between them, or one for importing concepts from one to the other. In the case of prototyping research, the premise is that design research and human-computer interaction look into tools from opposite sides – design research focuses on tool-using whereas interactive fabrication focuses on tool-making –, and that, within a design prototyping context, the one can be used to support the function of other. In both enterprises, tools are examined with regard to physical designer actions, and are what make actions and material manipulation possible. In design research, actions are recorded to be analyzed, evaluated in terms of their outcomes,
and, if proved efficient, to be used as methods in future practices. In interactive fabrication systems, actions are recorded in order to be translated into computer inputs to produce physical outputs. The method of this thesis is centered on the integration of frameworks that support both operations of action analysis and translation into computer input and output. This is where generative methods come to contribute. I use generative methods as the conceptual tool for translating results gathered through prototyping activity into computational tool inputs.

On the basis of the Active Prototyping framework is the identification of actions that can be recorded and extracted into information. Within the scope of the hypothesis that prototyping tools can be used as tools for active design exploration and evaluation, actions are viewed as events directly associated with the use and the capabilities of tools. To explore this hypothesis, Active Prototyping deploys the following operations:

(i) **Physical control of prototyping tools** that are connected to a computer system,

(ii) **Action recording** through continuous calculation of the position of the tool while it is being operated by the user,

(iii) **Exploration** of different design alternatives via the application of generative design rules to the recorded inputs, and

(iv) **Machine feedback** on the next steps of the prototyping process through means of automation or guidance of designer actions.

*Figure 9.* The four components of the Active Prototyping framework as they are synthesized in this research.
I propose Active Prototyping as a system for aiding designers to track their actions as they work on physical prototypes, and therefore to explore, archive and evaluate the design possibilities that these actions relate to. The technical implementation of this framework demonstrates how these operations relate to each other in action and within a physical context. These aspects have both a physical and a digital expression - physical in the sense of analyzing actions, and digital in the sense of recording, retrieving, processing them and bringing them back into the physical prototyping process. This duality creates the need for two parallel operations: a manual operation that allows designers to perform physical prototyping actions and a digital operation of tracking actions, extracting data from them, storing and processing that data, and making them available to designers in real time. However, this distinction is rather abstract; in a real-life context, Active Prototyping integrates these operations, and this integration is demonstrated through Fabcorder, the physical apparatus that implements it.

I develop Fabcorder as a prototyping device that implements the operations introduced by the Active prototyping Framework. The device consists of a work table attached to a hand-operated tool for material extrusion. A series of systems are combined to integrate the Active Prototyping operations:

- **Physical control of prototyping tools** is implemented through a two-state prototyping system for material extruding and gesturing.

- **Action recording** is implemented through a position tracking mechanism that is attached to the prototyping tool. The mechanism consists of three rotary sensors that are connected to each other through a system of taught strings. The sensors convert the angular position and motion of the tool to a digital signal. The digital signal produced from each sensor is communicated to an interface for shape calculation, display and storage. While the position tracking mechanism records movement in three dimensions, the work table can move across the vertical axis. The motor system that activates the table
communicates with the system that tracks and records tool position and movement, and therefore designer actions can be detected in three-dimensional space. By communicating the tool state described in (i) to the shape display interface, the system can record and represent not only the position of the movement in space, but also its effects to the physical object that is being prototyped.

- *Design Exploration* is implemented through a visual system for manipulating recordings and letting the user save, retrieve, re-use and re-design them as rules for generative design. Examples of such projections are transformations of an extruded profile which, stacked on top each other, form a three-dimensional prototype. The purpose of these manipulations and their projection is to define a range of design possibilities and outcomes after every step of the prototyping process, and to aid the designer choose among these possibilities in her next physical action.

- *Machine feedback* on the next steps of the prototyping process is proposed through two methods that are integrated into the action recording and the visual display systems.

### 1.3 Steps

This thesis is organized in the following way:

Chapter 2 will proceed with a discussion of human-machine schemes of interaction in prototyping environments and will analyze common concerns among *tool-using* and *tool-making* research attitudes.

- Section 2.1 will present the problem statement of this thesis, which lies in the observation that, among current inquiries, research on the development of embodied prototyping tools and research on hands-on prototyping activity are deployed as two remote enterprises.
• Section 2.2 will present and analyze findings from recent studies in design prototyping and will discuss how they contribute to the existing knowledge about (i) how different strategies for prototyping, and their associated tools, techniques, materials and workflows affect design decisions and (ii) to what degree these findings hint at flexible and efficient schemes of interaction between designers, design objects and prototyping frameworks. Focus will be given to the aspects of iteration, media and information gathering.

• Section 2.3 will discuss and analyze tools that have recently been developed through research in computational fabrication and in particular in the field of interactive fabrication. This section will discuss (i) what are the new types of designer actions and feedback that emerge through the development of these tools and (ii) what are the capabilities of these tools when used within a design environment.

• Based on the analyses in the previous two sections, section 2.4 will highlight ways in which the problem of informing design through making can be countered though new ways of using prototyping tools. In particular, this section will be centered on the problem of “lost information” between designers, tools and materials that occurs through prototyping processes, and will outline critical steps that need to be taken towards its solution.

Chapter 3 will introduce the thesis proposal, Active Prototyping.

• Section 3.1 (“Design vs making: A motivating problem”) will approach the questions addressed in the problem statement from the perspective of a specific design task that is common in architectural and engineering studios. This section will discuss that, while physical prototyping enhances experimentation with, and evaluation of, individual design solutions, due to its nature it doesn’t easily allow for exploration of multiple solutions as
frameworks for generative design do. Aiming for a prototyping scheme that is based on the interplay between physical experimentation and design exploration, I propose that combining systems for generative design with methods for analyzing designer actions and fabrication devices can bring physical prototyping tools and methods into a more exploratory context.

- Section 3.2 will introduce the fundamental principles of Active Prototyping. Focus will be given both on the analysis of capabilities that Active Prototyping introduces to methods for computational design and fabrication, and on the technical and material frameworks that can support it. Active Prototyping will be examined as the ability to develop a physical prototype without necessarily having a fixed design idea in mind, but instead, a series of ideas to test and select from – as the ability to design while making. In Subsections 3.2.1, 3.2.2, 3.2.3 and 3.2.4, the analysis of the framework components (physical control of prototyping tools, action recording, design exploration and machine feedback) will be illustrated in relation to the design problem described in Section 3.1.

- Section 3.3 “An articulated Active Prototyping workflow” will demonstrate how the principles introduced in Section 3.2 are applied towards the solution of the motivating problem described in Section 3.1.

- Section 3.4 (“Applicability”) will outline how the Active Prototyping framework can contribute to prototyping tools and practices that are currently being applied in education and the design industry.

Chapter 4 will introduce Fabcorder, the device that implements the Active Prototyping framework. Fabcorder integrates a variety of technical systems, each one corresponding to a number of the framework components analyzed in section 3.2. In this chapter I will outline future technical directions to which the proposed apparatus can be expanded.
Chapter 5 will present an analysis of the main features introduced
by the Active Prototyping framework and their potential to enhance future tool-making strategies and agendas. In particular, this chapter is going to discuss how action recording and generative methods can function as two novel additions to existing frameworks for computational design and fabrication, and to bring technologies for making into a more creative, human-centered and skill-based context.

1.4 Intended Contributions

This thesis aims to engage three kinds of audiences. For design researchers, it intends to problematize on current methods for studying, analyzing and enhancing designer actions through physical and computational prototyping processes. For researchers in human-computer interaction, it aims to provide new directions towards using human dexterity in the development of computational fabrication devices and in the design of interfaces serving creative human-machine enterprises. For makers and tinkerers, this thesis aims to demonstrate and share a technical framework for making machines that make based on physical inputs.
2. **Background:**

*Current trends in prototyping tools & methods*
2.1 Problem Statement

The natural sciences are concerned with how things are. Design, on the other hand, ... is concerned with how things ought to be.
– Herbert Simon, in The Sciences of the Artificial

The problem statement of this thesis is centered on the observation that, in contemporary inquiries, research on tool-using and research on tool-making are deployed as two remote enterprises. Based on this problem, this thesis questions how information extracted from prototyping activity can inform the design and use of computational prototyping interfaces and devices, and vice versa. This chapter frames the relevance of this question by reviewing methods that currently reflect it, as well as tools that are currently being developed to address it.

The chapter is initiated with a summary of how design research and research on computer-aided design, manufacturing and prototyping have been developed as two independent streams of inquiry throughout the past decades, besides exhibiting some critical interconnections. While the focus of the thesis is the distance between research on tool-using and tool-making as it gets expressed through contemporary practices, and how it can be resolved through new agendas, it is important to examine its roots in design as well as in CAD/CAM research, which has been, in the past decades, incorporated into the emerging field of HCI. The framing of the argument continues by illustrating contemporary examples that suggest how and why research on tool-using and research on tool-making should be integrated within common design and technological agendas. A review of the findings of current methods and of the capabilities of new tools indicates a yet unresolved need to seamlessly adapt technological tools to methodological research findings. Throughout this inquiry, what prevails as a critical issue that will be discussed in the following chapters is the problem of the “lost information” between tool users, materials, and tools themselves as it is manifested through contemporary practices, and how it calls for new schemes of interaction.
2.2 Trajectories in design research: Studying prototyping activity

In design research, focus has been given on the recording, reasoning and refinement of the different stages of the design process. The foundations of this stream were set during the 1950s by the work of mechanical engineer and later Professor of Design Research at the Royal College of Art, Bruce Archer, who aimed to establish design as an academic discipline by analyzing the design process as a dialogue between the natural world and a systematic model (Archer 1965) in which design goals and qualities are embedded. This view of design as an activity that can be investigated and directed through academic and manufacturing organizations (Bayazit 2004), as opposed to one that produces art and craft outside an institutional context, created a new generation of researchers studying the methods through which design gets conceived, prototyped and implemented within various contexts -academic and non-academic- and disciplines, for example engineering, industrial, and architectural design. Although in past decades, the main body of design research focused on schemes for design analysis and description (Stiny 1980; Gero 1990), more recent studies have shifted the focus on design prototyping strategies (Camburn et al. 2013), techniques (Buchenau et al. 2000), and dynamics (Dow et al. 2012). Recent studies have also contributed in adapting existing design production systems into a material context (Knight and Stiny 2015).

Within this realm of research, described from educationalist and writer Christopher Frayling as “research through design” (Frayling 1993), the implementation of experimental methods and empirical studies within design studios has shed light on how different aspects of prototyping activity enhance creative thinking, learning, problem solving and idea refinement throughout the design process. One conspicuous change in research studying how designers prototype their ideas is the transition of the disciplinary area in which it happens. Relevant research is currently developed within the fields of engineering, industrial design and interaction design; in architecture, it is not being developed extensively, as it is used to in the past through the work of pioneers as Donald
Schön and its dissemination in the practice of architectural design educators. Nevertheless, concepts as reflective practice and reflection-in-action, initially introduced by Schön, are prevalent among contemporary inquiries across all the aforementioned fields. Considering this shift, one of the goals of this thesis is to analyze critical aspects that constitute this reflection-in-action and articulate a scheme for introducing them into diverse means of practice, including architecture.

Among diverse empirical studies, three aspects prevail as highly determinant regarding the above features: (i) iteration style (parallel, serial or evolutionary) and number (Christie et al. 2012; Moe et al. 2004; Zemke 2012; Thomke & Bell 2001; Dow et al. 2009; Yang 2005), (ii) media used (physical/digital/mixed) (Bordegoni et al. 2009; Grønbæk 1990; Moten et al. 2014), (iii) information gained from developed prototypes (time spent on specific tasks, parts developed, use of software/hardware/tools) (Yang 2005; Camburn et al. 2017). Other aspects include scale, system representation and requirement fulfillment (Camburn et al. 2017). Considering the purpose of a prototype, which is to test a concept or process or to act as a thing to be replicated or learned from, these aspects can be translated as follows: (i) iteration can be translated into how we change a design; (ii) media used can be translated into how we implement a design; (iii) information gained can be translated into how we learn from this process of using both iterations and media.

A further inquiry into how each individual aspect from the ones discussed above contributes to the functional, aesthetic and pedagogical qualities of prototyping activities demonstrates more specific findings, explained in the following subsections. These findings contribute to the background of this thesis by providing insights on how prototyping activity contributes to the enhancement of the design process with regards to design exploration. Among these findings, what prevails as a critical issue is the role of the prototyping process itself, and how it affects not only design outcomes but also the designer’s engagement, perception and knowledge of the tools, materials and procedure use.
2.1.1 Iteration

*(how a design is changed)*

In design, engineering and interaction design research, findings have shown that iteration allows a gradual achievement of design goals through the development of multiple prototypes (Christie et al. 2012, Moe et al. 2004). Besides being critical to obtaining insights into specific construction and performance problems, iteration has been proven a useful procedure for identifying errors and simplifying object parts (Zemke 2012; Yang 2005). Studies have also shown that this gradual improvement can continue over many cycles of iteration (Moe et al. 2004). Throughout this process, a general observation is that the time to produce each iteration decreases significantly (Camburn 2017). Selection of fabrication methods has been also proved to have a significant impact on the number of pursued iterations (Thomke 1998; Yang 2013), and how time-efficient the prototyping process overall is (Ward et al. 1995).

A critical finding from current studies is the distinction between the different forms that iteration can take, and its impact on the functional, aesthetic and pedagogical efficiency of the process of prototyping. In recent inquiries, it has been shown that exploration of multiple design solutions at the same time enhances creativity and leads to more elaborate and designs. In engineering design, this practice is described as set-based design (Ward et al. 1995; Yang et al. 2013), as opposed to point-to-point design. In user interface design, it has been described as parallel prototyping as opposed to serial (Dow et al. 2010). These approaches add to the exploratory function of iteration by involving additional iterative steps within each prototyping stage. Based on this distinction, the thesis will explore how the strategies of set-based and parallel prototyping can be integrated in interfaces for interactive prototyping.
2.1.2 Media used

*(how a design is physically implemented)*

A common question among recent inquiries is how the media used during the prototyping process affect the final design outcome with regard to performance, aesthetics and cost. Research into relevant studies indicates a loosening of the barriers between physical and digital workflows for prototyping, as well as the acknowledgement that the right or wrong way to test a prototype is not always known, but dependent on what is the feature to be evaluated. Mixed prototyping techniques that involve both physical and digital media allow for this versatility. For example, digital simulations in conjunction with physical models have been proven to be an important tool for evaluating system dynamics (Moten et al. 2014). This conjunction has also proved useful in integrating different levels of fidelity within a prototype (Camburn et al. 2017).

The advantages of physical prototyping over digital, and vice versa, have been widely discussed. In summary, physical modeling has been proven more efficient in parts of the design process where decision making is required, as it provides the ability to assess the look and feel of the prototyped object (Smith 2000; Ward et al. 1995). On the other hand, digital modeling allows for precision and control of complex geometries (Oxman 2012), reduced time in implementing design changes (Sefelin et al. 2003), as well as communication with software and machines for computer-aided manufacturing. Considering the loosening of the barriers between the physical and the digital, and the rise of the need for mixed ways of implementing and evaluating design ideas, one of the big questions for design research seems to be how these mediums can be seamlessly integrated in order to simultaneously give designers a sense of accuracy, exploration and engagement throughout the prototyping process.
2.1.3 Information gained

*(how we learn from prototypes)*

The previous section discussed prototyping strategies, sequential and material-wise. But how are these sequential steps, these implemented techniques, and their associated impacts on the prototyping process measured? How can we improve our practices by gathering this information? As pointed out by recent state-of-the-art research on design prototyping, a clearer understanding of quantified information gained from prototypes is needed (Camburn et al. 2017). The term “quantified” in information related to design activity can have various meanings, as quantifiable actions or choices might have qualitative impacts. I take this observation to a more inclusive level with regards to the information gained, suggesting that, towards the improvement of current prototyping processes, a more thorough understanding of information, coming both from designer actions and their translations into artifacts through the use of tools -physical and computational- is essential.

The empirical studies presented in the previous subsections exhibit meticulous methodologies for gathering information related to prototyping activity. Such information can be the number of parts used (Yang 2005), user satisfaction (Evans et al. 2005), or time spent on a prototype (Elsen et al. 2012). However, there is little discussion on how these findings on tool-using can inform the development of new tools, or how the related information be gathered through the tools themselves -instead of only through empirical studies. The importance of these findings calls upon new tools that will be able to collect, and adapt to, data collected throughout the prototyping process.
2.2 Trajectories in human-computer interaction: New tools for prototyping

While the previous section focused on research on tool-using, and how it suggests ways in which prototyping processes can be improved through the articulation of the proper iteration, medium and information gathering strategies, this section will focus on tool-making. Within the scope of the thesis, tool-making is approached from a both digital and physical perspective. In particular, focus will be given on how emerging, computational tools for prototyping exhibit particular capabilities and constraints, and how the goals they set can potentially address the three components discussed in section 2.1 - iteration, medium use and information gathering.

Initiated within the field of computer science during the 1950s, research on computer-aided design and manufacturing aimed to aid designers systematize and control the design process, which would make possible the automation of the fabrication process. One of the main objectives of the CAD/CAM project was to free designers from the “toils of manufacturing and calculations” (Cardoso-Llach 2015). The way Steven Coons begins his description of the design process in his seminal 1963 paper titled “An Outline of the Requirements for a Computer-Aided Design System” epitomizes this objective: “The design process begins with a graphical description of a proposed device or system to satisfy a human need” (Coons 1963). This view of tools as facilitators of the implementation of a preconceived idea has considerably affected design practice throughout the past decades. The vast majority of the efforts described in section 2.1 use technologies that were developed under the CAD/CAM framework. As a result, design prototyping and making activities have been largely dependent on the capabilities of fabrication machines.

Recent research in human-computer interaction and engineering has made steps towards overcoming this status quo by introducing interactive frameworks for user engagement within the fabrication process. The concept of engaging the user as she is rapid prototyping an object has been introduced as interactive fabrication (Willis et al. 2011). The interactive fabrication framework is based on the principles of direct manipulation (Schneiderman 1983), however
adapted to a case in which interaction between the user and the fabrication machine occurs through an embodied interface (Willis et al. 2011). Such interface can be any tool that is connected to a computer system that allows users achieve their design objectives (Baudisch and Mueller 2017). The principles of the interactive fabrication framework are (i) continuous representation of the object of interest, with the focus of the user being on the fabricated form (ii) physical actions that are interpreted by the computer as inputs, (iii) rapid, incremental, reversible operations that are initiated through user inputs and produce outputs, and (iv) layered approach to learning across a variety of materials and mediums of expression. Essentially, what interactive fabrication suggests is an integration of hands-on approaches to craft with the operations and advantages offered by tools for computational design and fabrication.

Since its introduction as a concept in 2011, interactive fabrication has been instantiated in a variety of systems. The majority of these systems developed perform additive, subtractive or both additive and subtractive manufacturing. For example, Constructable (Mueller et al. 2012) supports precise and interactive laser cutting, enabling users to construct functional objects without the mediation of CAD software. Using the technique of wax coiling, D-Coil (Peng et al. 2015) allows novices to physically build three-dimensional models while modifying constraints in the digital model. Reform (Weichel et al. 2015) supports both material extrusion, milling and hand-shaping, while allowing users to reverse the last step in their prototyping process. On-the-Fly-Print (Peng et al. 2016) allows users to 3d print objects in low resolution while designing digital models, and also subtract material based on changes in the digital object using a cutting knife.

A critical observation about this new generation of tools is, according to Baudisch and Mueller, turn-taking: an interaction style in which the user produces some input, then the system updates the physical object, then the user produces another input, and so on (Baudisch and Mueller 2017). Turn-taking, according to the authors, is a being used as a result of the limitations of currently used fabrication machinery, while in the future it should be replaced by continuous means of interactive fabrication. This
vision resonates with the purpose of making fabrication tools for users that want quick results with minimal involvement of mediating software and design tools. In this thesis, I propose that the current state of turn-taking can be improved and expanded. The reason behind this argument is that this turn-taking supports the functions of (i) translation of designer actions into data during each stage of the fabrication process, and (ii) feedback on how design goals that have been set can be achieved based on the user inputs and the system outputs. To augment the fabrication process, interactive fabrication tracks user inputs in order to automate the output. In a scenario where automation can be replaced by skill, the thesis suggests that user inputs can be tracked in order to explore potential outputs and be stored for future use. This different view on the role of inputs and outputs implies new ways of feedback, and therefore interaction between designers and machines. The next section will argue how this approach on turn-taking can support needs expressed through current inquiries in design research.

2.3 Reclaiming the lost information between prototyping tools, materials and designer actions

The two previous sections described different directions currently being followed in design research and human-computer interaction. Among these directions, a major distinction was observed: in design research, a trend for analyzing the efficiency of the prototyping process through use of tools; in human-computer interaction, a trend for making prototyping tools capable of guiding and automating the design process. What makes these approaches different is the attitude towards the relationship between the user and the tool: in design research, focus is given to how the users (designers) use their knowledge in order to achieve design goals through prototyping; in human-computer interaction, focus is given to the capabilities of the tool itself, and how these capabilities are perceived and harnessed by the user. What is common among these two approaches is the exchange of information between users (designers or non-designers) and tools; in the tool-using scenario, information needs to be recorded and extracted, whereas in the tool-making scenario information needs to be translated into system output.
This thesis views this distinction as an opportunity rather than as a barrier. The findings and systems analyzed in the last two sections justify how concepts currently developed under strategies for tool-making can enhance research on tool-using. A particular opportunity in this potential collaboration is that of using devices for interactive fabrication towards recording, analyzing and evaluating designer actions throughout the prototyping process. The next two chapters will articulate frameworks and systems that address this potential and support the hypothesis that prototyping tools can be used as tools for design exploration and evaluation. Based on generative methods, these frameworks and systems will propose ways for exploring design while making through interfaces for design transformation, projection, selection and implementation based on physical inputs.
3. Active Prototyping: Informing design through making
3.1 Design vs Making: A Motivating problem

You can find out how to do something and then do it or do something and then find out what you did.
— Isamu Noguchi

“Design,” “making” and “prototyping” are terms that carry different meanings and uses in various disciplines and industries. Tools, systems and final products might differ in each field, but the terms have a similar relationship with each other. Design is to plan, whereas making is to produce, and prototyping is to evaluate a design through making. The established definitions of the terms express this generality: design is “to create, execute or construct according to a plan,” (Merriam-Webster, 2018) whereas making is “to bring into being by forming, shaping, or altering material.” Prototyping is “to form a prototype,” which is defined as “a preliminary model of something, from which other forms are developed”. Prototyping is essential to design because it introduces making. It is interesting how in disciplines related to design, these terms keep their general meaning: in architecture, engineering and industrial design, “design” relates to how a design idea is being developed, “making” refers to how this same idea is implemented, and “prototyping” relates to further developing and testing, essentially “informing” this idea through making.

The paradigms in Chapter 2 instantiated two prototyping approaches for informing design through making: one centered on the use of tools, and one centered on tool development. What prevailed as a critical step towards enhancing the implications of these studies in contemporary design practices is the focus on designer actions; in particular, the collection of information about (a) what makes these actions effective –based on their outcomes-and (b) how efficiency of actions can be augmented through new tools. Research in the first approach is based on the observation of the process and its outcomes as complete artifacts, whereas in the second approach, systems were based on an incremental exchange between actions translated into inputs and outputs. The question that I make through analysis of both approaches, and I addressed through the hypothesis of the thesis, is how informing design through making can happen in real time.
This hypothesis can be instantiated in various design examples, and is independent of tools, techniques and materials used. The example analyzed below brings this hypothesis into a more practical context. Let us imagine the task of designing and prototyping a three-dimensional, free-standing object that needs to have a certain height (h), width (w) and length (y) to allow it to carry the weight that it supports. If the object is a pedestal, it should be able to carry the weight of the objects that are supposed to be placed on top of it; if it is a column, it should be able to afford the weight of the elements (e.g. roof, floor) that it supports. Let us start with an empty bounding box, and speculate on what form parameters a designer might consider when designing this object (Figure 21): what are the boundaries of the object thickness? What are the possible profiles that can be used in the cross-section of the object - how will it look like if this cross-section were a square, or freeform profile; what if this profile varied instead of being uniform across the z-axis; if so, what would the logic be behind the transformations creating this variance?

Figure 20. Fabclay, a project exploring clay printing in architectural scale with columns as a design problem ('FABBOTS 3.0' Research & Development Studio, IAAC, 2012).

Figure 21. The column problem, which will be used as a motivating example for the Active Prototyping framework.
These few form aspects are enough to generate a myriad of possible design solutions, even with short sets of sample widths, lengths, profiles and logics of transformation. One could just sketch her own solution and then design it without exploring even a part of this range of possibilities. But let us think of a case in which exploration of possible solutions is needed - which is typically the case in design education and practice.

In this case, a series of schemes and tools for generative design can be used to visually populate multiple possible solutions. Approaches to this design problem can vary depending on the system that is used to generate this range. The three following examples demonstrate how visual design exploration can be implemented through a variety of computational procedures. Later on, this analysis is going to transferred into a context of making. In particular, I will demonstrate how the type of exploration that visual ways of representation offer outperform, and respectively, fall behind, physical exploration that prototyping can offer. But first, let us focus on the design problem discussed above.

A computational way of enumerating the range in question is using the formalism of Shape Grammars, an example of which is shown in Figure 22. The parameters of the column problem (h, l, w and p) can be embedded into a grammar that contains a simple set of shapes (ps), rules (applied across h), and transformations that affect both shapes and measurements (h, w, l) (Figure 23).

![Figure 22](image)

**Figure 22.** A computation for producing the design on the right, starting with the rectangle on the left as the base shape and using the transformation shown in the middle shape as the rule.

In this scenario, profiles (ps) can be distributed across the height (h) axis using rules. Within these rules, they can be manipulated through transformation operations, for example scale, rotation, etc. Figure 24 demonstrates how, four simple rules (addition, subtraction, division and scale) can embed the parameters of the design problem and produce a set of solutions within a designer-
defined range. Assuming that we want to distribute profiles across the axis of the given height, the height of the object’s parameter-defined bounding box needs to be divided (division rule), then abstracted into planes (subtraction) to be modified (here by scaling, but different transformations might apply) and finally be merged together (Figure 25). Given a simple set of \( n \) possible values for each one of the object parameters, \( 4^n \) column designs can be generated.

**Figure 23.** Example transformations used by the shape grammars scheme.

**Figure 24.** Transformations used in the distribution of the profiles across the z-axis of the column bounding box.
The shape grammar scheme is used as a formal, computational way of approaching the problem. In a computer-aided design system, the logic for exploring possible solutions would be similar, though the implementation procedure and mediums would be different: rules would be replaced by commands such as “move,” “rotate,” “scale,” “mirror” etc. (Figure 26). Using the given h, w, l, and p values/types as inputs, the same number of cross-referenced designs can be produced. A similar procedure would apply using a computer script (Figure 27).

These generative approaches differ in terminology, medium, intuitiveness, and most probably enumeration time. However, when it comes to implementation through prototyping and making, they exhibit the same problem: when the exploration, or search, is complete, all a designer has is a number of designs, each of which corresponds to a separate iteration. Thus, in order for all of the designs to be evaluated, they need to be fabricated separately - all of them or a number of them. Let us not count the time that this process might take, but rather focus on what might render work that has been done in the design stage invalid. In this case, not only will we have spent time to design and fabricate all -or a number of- these artifacts, but we will also have to go back and re-think, re-design, re-fabricate them. This parameter is change: change in a design decision or change in tools and materials that are going to be used to prototype this object. For example, based on the column problem described above, let us imagine that we
have gone through the workflow of prototyping an object only to realize that the profiles are too sharp in real life (change in design decision), or that the tools that are in the designer's availability cannot achieve a particular detail developed during the design stage (change in the use of tools). How can this change, its inherent role in the design process, and the further aspects to which it is associated to, can be embedded in such a generative scheme? In other words, how can design exploration be informed through making, while making?

The precedents analyzed in Chapter 2 take a step towards answering this question. However, some important aspects of it still remain unanswered: these parts, as Chapter 2 concludes, are (a) gathering information from designer actions as they are being developed, a remaining question mostly addressed through experiments in design research— and (b) investigation of means of feedback that designer actions exhibit, and the design possibilities that they allow for. If schemes for generative design provide methods for enumerating and visually exploring a variety of solutions before making, and tools for prototyping provide a direct way of physically evaluating design outcomes after a design has been defined, how can design be evaluated while making?

Figure 27. Computer script generating the column design.
3.2 Solution: Active Prototyping

I propose Active Prototyping as a framework that introduces a new range of capabilities in tools for computational design and fabrication. The idea motivating the Active Prototyping framework is that, by collecting information related to the use of tools during the development of a physical prototype, designers can archive the design knowledge that these actions are embedding, and by further processing this information through schemes for generative design, they can explore, control and improve design outcomes. Essentially, Active Prototyping is the ability to develop a physical prototype without necessarily having a fixed design idea in mind, but instead, a series of ideas to test and select from. In this framework, action recording is the fundamental process around which a series of operations are deployed: it implies the use of systems that embed both prototyping tools and action recording mechanisms. Additionally, it requires systems for storing and processing these recordings, as well as for bringing the results of this processing back into the making process (Figure 28).

**Figure 28.** The four components of the Active Prototyping framework and their interaction.
For example, in the column problem described in the previous section, Active Prototyping provides a solution for evaluating the distribution of different profiles (ps), each one having its own w and l, across the height (h) of the object’s bounding box, using rules and real actions. Such actions can be actions related to building profiles, or actions related to the exploration of how these profiles can be transformed and distributed across the z-axis using rules and transformations. Therefore, the goal is to capture actions as a three-dimensional movement in space, but also to record their effects on the physical prototype. In a general sense, a framework that combines a physical operation of performing prototyping actions with a digital operation of processing and facilitating these actions, is needed. If we examine how this general concept translates into a real design problem, a series of more specific questions arise. For example, how would the shape of a starting (base) profile in the bottom of the object affect its ability to be transformed in the next steps of the prototyping process, and create new profiles? How will these shapes look if converged into a single geometry? How will the physical and digital tools that are being used allow for this exploration?

These questions call upon an inclusive examination of what a single designer action comprises within a physical prototyping process, how sequences of single actions are deployed to physically implement an idea, and what computational procedures can be instilled within these sequences in order to allow design exploration parallel to the physical prototyping process. Towards this goal, I introduce Active Prototyping as an integration of the procedures of tool use, action recording, design exploration and machine feedback. In the following subsections, I synthesize these procedures while providing a description of how they would be used within the workflow of the parametric column design problem. I demonstrate tools and functionalities in a schematic way in order to articulate the conceptual and operational components of the framework. In Chapter 4, I demonstrate how these functionalities are implemented through Fabcorder, a technical apparatus that implements the Active Prototyping framework.
As a prototyping method, I introduce a technique for gesture and movement-based additive manufacturing. I introduce this technique as a direct way for recording, processing and reflecting on designer actions that occur during the development of a physical prototype. After the introduction and analysis of the fundamental Active Prototyping Functionalities, I demonstrate a complete workflow for implementing the column problem that I use as a motivating example.

### 3.2.1 Physical tool use

Active Prototyping is based on the recording and computational translation of actions that occur while the user works on a physical model. The goal of action recording affects the design of the prototyping tool itself, as well as the way in which it is going to be used on the part of the designer. Therefore, the design of tool itself, and its capability to be recorded, is the first step in the Active Prototyping framework.

In order to be recorded, tools need to be connected to a system for motion or position tracking. This implies that, in the case where tools are used in different ways, the state in which the tool is being used, for example “on” or “off” needs to be communicated to the action recording system. Additionally, the fact that the tool is connected to a tracking system implies certain limitations to the spatial and the functional range within which it can be used. I translate these needs into the functionalities of affordance embedding and operation initialization.

- In a system that is based on the recording of actions, it is important to examine how an action is defined, what different types of actions can exist within a prototyping process, and what are the physical limitations that these actions relate to. I refer to this procedure as embedding of affordances. Affordances are here referred as actions that are possible, discoverable and perceivable (Norman 2013) through the use of a physical apparatus. An existing example
of associating tool affordances with a prototyping apparatus is Constructable (Mueller et al 2012), an interactive fabrication system that offers a number of hand-held proxy laser beams that result in a different fabrication shape or operation, for example drawing standard or freehand shapes or bending surfaces. In Active Prototyping, affordance embedding is communicating to the user, through the design of the prototyping tools, what actions are possible by the tool, and what is the possible physical range related to the development of these actions (Figure 29).

• Operation initialization is letting the system know what tool is being used and in what way, so that this information will be reflected in the recorded result. An example is using the tooltip of a material extruder in an inactive state as a tool for gesturing instead of extruding material. Another example is using the tooltip of a cutting tool as a tool for marking or carving instead of subtracting material. This functionality creates the need for user input before and after a prototyping action is developed. In a system that is based on the recording on physical actions, it is important to feed this kind of input through the physical interface of the tool. Figure 29 shows a way for providing this input to the system in such a way.
3.2.2 Action recording

Action recording is the conceptual and functional basis of the Active Prototyping framework. Actions can be either prototyping actions that have a direct effect on the physical prototype, or gestures that have a more speculative, exploratory character and explore the prototyping effects that they would produce if they were applied to the physical prototype. Through being recorded as the designer operates the tool, both types of actions can be translated into computer inputs and be further analyzed and manipulated. This operation is useful both regarding the analysis and “saving” of prototyping procedures and also with regards to introducing an additional feature into tools for interactive fabrication: that of not only controlling (Willis et al. 2013) fabrication devices but also recording, analyzing and archiving ways in which they can be controlled.

Selecting the proper position tracking system for the purpose of recording designer actions is a crucial step, and strongly depends on the purpose of the prototyping application and the design of their associated tools. Current motion tracking systems are broadly categorized into optical and non-optical, with non-optical being sub-categorized into mechanical, magnetic, acoustic and inertial. An overview of interactive fabrication systems indicates that each method for motion tracking exhibits different levels of compatibility with different fabrication devices. For example, D-Coil (Peng et al. 2015) uses a non-optical (magnetic) tracking system for position sensing, allowing an additive manufacturing device to freely follow the user’s hands in three-dimensional space. Shaper (Willis et al. 2011), also an additive manufacturing device for interactive fabrication, optically translates gestures created through user motions into to shapes.

In Active Prototyping, actions are recorded for the purpose of being digitally processed and then physically guided or automated. Thus, the selection of a motion
tracking mechanism that can also be used as a feedback and action-performing mechanism is essential. Towards this goal, I introduce a recording method that can be used as a prototyping system as well (Figure 30). In this method, the tool is attached to a non-optical system that can be used as a recording device when operated by the user, and as a performatory device that can replicate or automate shape inputs. An implementation of the basis of the technical framework supporting this mechanism is demonstrated in Chapter 4. The performatory function of the mechanism is mostly useful in the case where the tool is used in its “on” state as a material extruding mechanism.

### 3.2.3 Design exploration

The first two steps of the Active Prototyping framework contribute to the initialization and the recording of designer actions during the prototyping process. Until this stage, Active Prototyping serves the purpose of information gathering from a prototyping procedure; essentially, it is not yet active, as it does not yet provide a framework for design exploration, selection and
evaluation. What introduces these aspects is user interfacing: enabling the user to retrieve, reuse and re-define a design while making.

User interfacing allows for design exploration by bringing stored actions into a generative design context. In particular, it allows the user to project multiple design ideas and to select among them before moving on to the next prototyping action. This need does not apply in a digital design context, where design solutions can be explored by assigning different variables such as dimensions, properties and transformations to model elements and have a range of solutions produced at the same time. For example, in the column example, multiple designs can be produced at the same time using a set of elements, variables and transformations that are embedded in the same generative procedure. In a physical context, exploration of multiple designs while working on a prototype depends on the physical changes in the prototype. In Active Prototyping, I overcome this limitation by introducing a way to explore designs by projecting hypothetical design actions on a digital representation of the object that is being prototyped.

This operation implies that (i) recorded actions need to be directly retrieved so that they can be manipulated, (ii) given that the user might want to manipulate a stored physical shape without repeating it, the system needs to provide a digital way for implementing shape transformations. I implement these goals by proposing a visual interface for shape display, storage and manipulation that communicates with the action recording system proposed in 3.2.2. The system receives the input from the motion tracking system and offers the user the option to manipulate shapes using transformations as rules. These transformations can be applied either physically or digitally as follows: given an input shape \( P \) that has been extruded at a height \( h_0 \), the user can move the tool in the position \( h_1 \) and explore how an extrusion based on shape \( P_0 \) will result in the following ways (Figure 31):
• Through physical gesturing: by creating a new shape \( P_1 \) using the motion tracking mechanism, and the extruder in its off state, and then projecting this transformation on the digital representation of the object.

• Through the interface: by applying transformations such as translate, scale and rotate to \( P_0 \), controlling the associated values of these transformations, and projecting them on the digital representation of the object.

Figure 31
After transforming a shape, the user can project its effects on the physical object through means of interfacing. By moving the tool across different physical positions and projecting shapes, the user can visually test a variety of designs. Figure 32 shows such an exploration.

### 3.2.4 Machine Feedback

I propose machine feedback not as the closing step of the Active Prototyping framework, but as one that completes the loop of each prototyping action, its recording, and its exploration through the proposed interface. This is the step in which the system is used to facilitate the implementation of design solutions that have been priorly explored and selected by the designer.

I propose a system for machine feedback that facilitates the physical implementation of designs in two ways: (i) visual and tactile feedback on designer actions, in which the interface guides the implementation of a selected shape and (ii) automation of selected actions, in which the system performs the selected action on behalf of the designer:
• I propose visual machine feedback through physical use of the tool. In this case, the interface for shape calculation visually informs the designer on the accuracy of the performed action as she operates the tool across the selected toolpath (Figure 33).

• Using an action recording mechanism that can also serve as a performatory system offers the opportunity to use the initially manually controlled system as a system for automating designer actions without physical interaction on the part of the designer.
3.3 Designing While Making: An Active Prototyping workflow

Figure 34 demonstrates a schematic diagram of a complete workflow implementing the motivating problem described in section 3.1 through the Active prototyping framework. The diagram demonstrates how the four fundamental operations of the Active prototyping framework are combined in an exploratory additive manufacturing procedure. In particular, the diagram demonstrates how physical tool use is combined with action recording, how the data collected from action recording are used in a generative process of computational design exploration, and how the outcomes of this exploration are brought into the physical prototyping process through means of machine feedback.

The process is developed as follows:
In frame (a), the user initiates a material extruding operation in the system, and enables recording.

In frames (b) and (c), she performs an extrusion while recording her action. The shape formed during the action is calculated and stored in a system for visual display.

In frame (d), she moves the tool to the vertical position $h_1$ from position $h_0$, and performs a spatial transformation to the shape that she originally extruded. The transformation is applied digitally, and applies as a rule in relation to the initial shape.

In frame (e), she continues her exploration by projecting different transformations in various $h$ locations. At the same time, these transformations are projected on the interface for shape calculation.

In frame (f), she selects an action from the ones projected in step (e).

In frame (g), she automates the implementation of the action before moving into the next physical action, which moves on to an upper part of the prototype.
Figure 34
3.4 Applicability

I proposed Active Prototyping as a framework that can be introduced into the workflow of further tools and techniques than the one that is being demonstrated. From a user perspective, Active Prototyping, as I described in the previous section, Active Prototyping can serve as a medium for hands-on design experimentation, exploration and evaluation. An additional level of functionality arises when we think of Active Prototyping from the perspective of a design researcher or a tool-maker; in particular, if we think of it as a tool for recording designer actions and collecting information related to the use of a tool and the effects of this use into the prototyping outcome. In this case, Active Prototyping has the potential to be used as a framework for recording and testing aspects of tool use.

A number of the studies that were discussed in Chapter 3 can benefit from this use, both existing and new tools and technologies for making. One of the issues that directly arises when introducing the Active prototyping approach to any given workflow is the level to which the action recording process limits the capabilities of the tool use, and thus the range of capabilities that the designer can exhibit while using it. In the example demonstrated in the previous section, we saw that the use of a particular technology for motion tracking constrained the use of the tool to a specific range. For example, in experiments in autimotive industry, we see that tools for hands-on, clay prototyping are often preferred instead of less "messy" mediums (Ward et al. 2015). If the efficiency of this method directly correlates to the freedom that the prototyping tool provides to the user, it is important to consider how compromising tool capabilities for the purpose of motion tracking can affect the quality of research results. Towards this goal, further inquiry into efficient and less-contraining means of action recording is essential in future inquiries.
4. Implementation: 

*Fabcorder*
4.1 Functionality

Each one of the four operations introduced in the Active Prototyping framework is related to different technologies. In particular, action recording is related to systems for motion and position tracking; this implies that tools involved in this process should be physically or digitally connected to these systems. Moreover, visual exploration of design solutions requires systems for shape display and manipulation. Taking the outcomes of this exploration back into the making process requires technologies for visual and tactile feedback.

I introduce Active Prototyping not only as a conceptual framework but also as a strategy for bringing such technologies together through novel tools for design and making. To demonstrate the Active Prototyping framework, I develop Fabcorder, a technical apparatus that implements a number of these operations. The name of Fabcorder comes from the words fabrication and recorder: it is a machine whose functional basis is recording of designer actions. Fabcorder is a novel system for additive manufacturing that can be expanded to other prototyping techniques, for example subtractive or formative manufacturing.

The translation and integration of the operations introduced in the previous chapter—physical control of prototyping tools, action recording, design exploration and machine feedback—into physical apparatuses and computational interfaces are two not only technical, but also conceptual, challenges. The reason for this is that the technical integration of these systems affects the design of the produced system, its design and making capabilities and, most importantly, its interaction with the user. In the case of Active Prototyping, the operation of action recording can limit the application of the tool in two dimensions, and can also hinder the use of tools that have a less predictable result, for example the use of a brush with a non-uniform outcome, or the use of a sculpting tool that can produce asymmetrical cuts.

I consider these challenges as important questions to explore in future work, and introduce techniques for recording while making within the spectrum of an application whose results are
relatively controllable. In particular, I design a pneumatic material extruder that can be manually operated and, at the same time, traced in space by being physically attached to a position tracking mechanism. As for the limitation of two-dimensionality of action recording, I introduce a system for enabling the implementation of designer actions in layers, and thus, three dimensions. The system for design exploration that I design and implement is based on the operation of the layer-based recording mechanism. I propose the implementation of feedback, an operation not supported by the current implementation of Fabcorder, as an operation that is compatible with the workflow of the implemented aspects.
4.2 Technical description

Fabcorder consists of a working table with an attached mechanism for a tool receptor. The tool receptor is connected to a mechanism for motion tracking. For motion tracking, I use the method of position monitoring using the shaft of an absolute rotary encoder, a practice common in electro-mechanical applications, and especially in machines for computer numerical control (CNC) and robotics. For shape calculation in 2-D and additional precision, I use three rotary encoders that are placed in the periphery of the working table. The tool receptor takes tools for additive manufacturing, in particular a cartridge capable of extruding clay using compressed air. The apparatus performs motion tracking through a system of three strings, each one attached to the tool receptor and the rotary encoder. The three encoders are connected, through a hardware system, to a microcontroller that reads the three signals generated by the move of the tool tip moves across the x-y axis, and converts those signals to absolute position coordinates using a trigonometry algorithm.

Essentially, this system performs the opposite operation of which a CNC machine or an XY 3d printer does (Figure 36). In current rapid prototyping tools, machine operations are made possible through motor control. In Fabcorder, I introduce a novel way

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Figure 36

Active Prototyping
of using this technology in order to track designer actions when using a prototyping tool as a physical tool, and propose directions in which the same tool can function as a digital tool through means of automation. Instead of using the system shown in Figure 36a as one for controlling motion and thus performing actions, I additionally use it as one for recording motion (Figure 36b), without depriving it from its ability to perform actions.

Through a motor-gear system operated by the user through a button, the x-y mechanism can be adjusted to different heights. Two or more shapes in different heights can build layered forms in 3-dimensional space. The task for the microcontroller is to determine, in each layer, the position of the tool tip, given the length of the three taught-strings to the three separate anchor points and the position of the x-y table. By collecting a set of points after the completion of an action, the system represents and archives the performed shapes in a 3-dimensional computer environment. This is implemented by connecting the microcontroller that receives the signals from the rotary encoders and the motor system to a digital interface for shape representation (Processing IDE). By digitally manipulating the calculated shapes using the interface, or by moving the motion tracking mechanism without activating the extruding operation, the designer can reflect on a range of possible design outcomes after completing a step of the prototyping process, and to choose among these outcomes.

An axonometric view of the Fabcorder system and its main components is shown in Figure 37.
Extruder on / off switch

Action recording

Generative design Interface

Figure 37
4.3 Integrating frameworks

In the following sections I describe the design and the technical implementation of the operations of tool control, action recording and user interfacing, and outline methods in which this implementation will include the operation of feedback in future work.

4.3.1 Physical tool use: Extruding/gesturing

Physical tool use is implemented through a two-state system for clay extrusion (Figure 38). When in the print state—which can be defined through a physical control panel placed on the frame of the device (Figure 39)—the user extrudes. When in the draw state, the user records her gesturing and saves it as a shape for future use or experimentation. Using the up and down buttons, the user can move the adjustable table mechanism in different position and therefore print, draw shapes and perform spatial transformations in different coordinates.
4.3.2 Action recording: Calculating tool position

Action recording is the core operation around which the technical implementation of Fabcorder is developed. I implement action recording through a novel system for continuous shape calculation of a hand-operated tool, which in the case for the Fabcorder is a penumatic material extruder. Figure 40 shows Fabcorder without the material extruder attached in the intersection of the wires through which I implement shape calculation. An important feature of the motion recording mechanism is that, through the adjustable table shown in Figure 41, designer actions can be recorded and saved in three-dimensional space.
The shape calculation procedure is implemented as follows: the tool is initially placed at the center of the table. As the user moves it across the x-y plane, the retractable wires, the one end of which is attached to the base of the device and the other to the tool receptor, move the shafts that are placed in the periphery of the wooden frame. The shafts, in turn, cause a turn in the shaft of the rotary encoders that are attached to them. By generating 1024 pulses per rotation, the rotary encoders are very sensitive to movement and thus can catch even subtle movements of the tool across the x-y plane. In each frame that the tool is moved, a pulse is generated. The two important components of this calculation are:

- the conversion of each pulse into a distance $x_1$, $x_2$ or $x_3$ as shown in Figure 42. This is implemented through multiplying the angle by which the shaft has been rotated by the length of the periphery of the
The second important step in the calculation process is the calculation of the absolute position of the tool based on the three received lengths. I solve this problem using a simple trilateration algorithm shown in Figure 43. Figure 44 shows, in a sequence of frames extracted from a continuous action, how the inputs received by the rotary encoders are used to calculate and represent the coordinates of the performed action in the computer interface.
4.3.3 Design exploration: Intefacing

In the previous section I elaborated on the implementation of action recording through position calculation. The next step in the Active Prototyping framework is design exploration through means of interfacing. Figure 45 shows the interface of a proof-of-concept application built for the purpose of processing the inputs received from the action recording process. By moving the adjustable mechanism in different heights, the user can project multiple profile transformations based on a base shape. The basic transformations that are supported by this interface are rotation, scale and translation, and using the sliders, the user can assign different values to these transformations; essentially, to In Figure 45, this base shape is a rectangle. Future implementations of the interface will integrate the manipulation of any shape registered into the system through the action recording mechanism.
The current implementation of the interface is structured as follows:

The user inputs a shape through the motion tracing system. Gesture-based shapes are represented as outlines, while extruded shapes are shown in solid color (Figure 46). The transformation sliders on the right provide the user the option to perform the spatial transformations of scale, rotation and translation across the xy plane. The user can project these transformations in different heights through using the up-down switches that control the motor system. As the motion tracking mechanism does not move in the z axis, in order to extrude or project a shape at a certain height, the user has to lower the table. When the table lowers by a level, this means that the "current" level of the interface increases (Figure 47).
After adjusting the table to the desired height, the user can perform transformations on the recorded shape and project their effects on the object (Figure 48). Essentially, the interface aids the user visually explore curve interpolations in 3-dimensional space. An additional feature of design storing is available so that the user can "save" designs for future use or comparison with other designs (Figure 49). Figure 50 shows how the current interface follows its conceptual description in Chapter 3.
4.3.4 Feedback: Directions

Development of methods for feedback is the next step in this research. Based on the proposed technology for action performing and recording, as well as its integration with the interface for visual display, I will continue this work towards two directions:

- Sensory feedback, which can involve either visual or tactile feedback on the facilitation of a design action that has been selected during the interfacing process. This kind of feedback can be useful in the case where the designer needs to practice a prototyping technique.

- Feedback with the absence of interaction between the designer and the tool. This kind of feedback essentially translates to automation, and it can be useful in the case where a challenging prototyping action or a repetitive process needs to be implemented.

These directions consist the main goals for future work for this thesis, together with additional directions discussed in the Discussion chapter.
5. Discussion & Conclusions
This thesis started with a problematization on how strategies for tool-making in computational fabrication can integrate the need for hands-on engagement, experimentation and learning with physical prototyping. Based on the observation that physical prototyping enhances experimentation on a single model, while schemes for computational design enhance exploration across a broad range of solutions, and envisioning a framework for design exploration while physically making, I looked into computational design methods that can expand the range of this experimentation and systematize its process. I used generative methods not as a solid existing background to project onto prototyping tools and methods, but rather as broad concept to learn from and to adapt to a material context, considering that the majority of current research on generative methods is implemented in digital design scenarios.

I proposed Active Prototyping as a conceptual framework and as a practical workflow for integrating this generative approach into future prototyping practices. I proposed that, in order to serve as a better medium for expression, prototyping must be active besides rapid: active in the sense of integrating our actions and design decisions as we prototype. In Chapters 3 ("Active Prototyping: Informing design through making") and 4 ("Implementation: Fabcorder") I proposed a framework for rendering this process active, and introduced a conceptual workflow of interaction with tools and a technical apparatus that implements critical aspects of this workflow. On the center of this framework and its associated tools and workflows was recording, which consists the main contribution of this thesis, and shapes its agenda for future work:

- recording as a result, referring to the a posteriori collected data of a design process and how they can be analyzed and reused,

- recording as a process, referring to the data collected and processed while a design and prototyping process is being developed, and how they are used to enhance the process itself,

- recording as a research tool, referring to how, not only as designers but also as tool-makers and design researchers, we can learn from the way that other designers and makers design, prototype, explore and materialize their ideas.
These three axes create a rich discussion that is directly related to the goal of the Active Prototyping framework, which is design exploration, and the conceptual apparatus that I use towards this goal, which are generative methods. These axes also relate to the low-level details, such as tools and technological systems that make these bigger goals possible. In the following subsections I will discuss these three novel aspects of recording that emerge through Active Prototyping, and I will outline future directions in which they can enhance contemporary design and fabrication practices.

5.1 Recording as a result

Recording as a result prevails as a side but important feature of the Active Prototyping framework. With the appropriate storage space and access interface, recording translates to storing. In contemporary tools for design and fabrication, focus is given on developing new actions based on an initial drawing, prototype or model. Interfaces enable copying, undoing and saving multiple versions of designs. Out of these many designs, only a few are selected in later design stages; usually, the question of why a design is rejected is not asked. Through recording and further analysis of the actions that resulted in “good” or “bad” designs, designers can not only identify what worked and what didn’t work for them, but most importantly, what design actions led to these results. Additionally, successful workflows can be saved for future reference and use.

This potential use of recording can also benefit robotic applications and Artificial Intelligence systems for design and construction. Although AI is a topic that was not discussed within the spectrum of this thesis, future work initiated through the Active Prototyping framework can provide answers to currently asked questions: what makes us humans creative, and, at the same time, capable of reasoning? What aspects of our mental and physical activity enhance our design thinking and our manual dexterity?
5.2 Recording as a process

While recording as a result has the potential of informing future actions, recording as a process has a value related to the now of making, for example how do we “save” an action to test another one, and then get back to it to compare the two of them, or how we improve a result through prototyping. Recording as a process was the main recording objective of Fabcorder, which demonstrated a proof-of-concept of this functionality. The goal in Fabcorder was to perform and project transformations of recordings, and ultimately get machine feedback on the next steps of the prototyping process.

In this human-machine collaboration framework, what prevailed as a critical component is to what extent our actions can be recorded in real time when we are using a tool with our own hands: how can a system measure not only the exact position of the tool, but also the way we hold and use it, the inconsistencies in our hand movements, and most importantly, their effects on prototyping outcomes? In a system that assists a process, effective and inclusive means of representation are essential in both sides, the human and the computer. On the human side, recording is storing, accessing and modifying, whereas on the computer side, recording is representing accurately, remembering and updating. In this case, recording requires new technical systems for real-time information exchange and collaboration, but also novel theoretical insights on effective means of human-computer interaction.

5.3 Recording as a research tool

In Chapter 2, the majority of the prototyping studies that were presented, either they examined the use of existing prototyping tools or newly introduced tools for interactive fabrication, studied prototypes as results. Many of the studies collected information on aspects of the prototyping or user testing activity that related to the process (for example, number of prototype parts that were used, or time elapsed between the implementation of two tasks), however focus was given on the results of the prototyping process. This is reasonable in the sense that design is being judged by its results. Rather than arguing against this approach, I propose that,
through recording, the findings of these studies can be further enriched. If observing and judging by the results can be subjective or coincidental, recording can make this process more systematic, valid and accurate. An example application of this hypothesis can be studying designer actions when using a tool or a prototyping technique through means of recording rather than through moments in the process of the experiment or its prototyping results.

In all three cases, and more that may emerge through future inquiries, recording itself is nothing without a system for analyzing, organizing, processing and displaying actions and their results. In Active Prototyping, this system was generative methods. Due to their ability to be used both as schemes for analysis and schemes for design and information generation, generative methods have the potential to support all the three functionalities of recording that were discussed above.

Through Active Prototyping, recording prevailed as the main mediator between human actions and machine feedback. This thesis took a first step in exploring this relationship, its potentials and its dependencies on the technologies that are being used. The conclusion from this discussion is that, in a human-machine collaboration framework, what can be recorded can be translated into computer input, and consequently be computationally augmented. The challenge, in future work, lies in how we perceive and design what is to be recorded, and how this perception can enrich what we want to achieve through this recording.
6. Contributions
This thesis started with the hypothesis that tools for physical prototyping can be used as tools for active design exploration and evaluation. I proposed that, towards this goal, the integration of prototyping methods with methods for generative design, is essential.

I introduced Active Prototyping as a computational framework that enables designers to:

- Physically engage with design objects during the process of prototyping.

- Record their actions and their translations into design outcomes, a functionality currently not supported by tools for physical prototyping and interactive fabrication.

- Explore a range of design ideas through the application of methods for generative design, and choose among these ideas.

- Get feedback and assistance on the physical implementation of designs.

I developed Fabcorder, a prototyping apparatus that implements a number of the above operations. I emphasized on the design and technical implementation of a system for action performing and recording, which is the conceptual basis of the Active Prototyping framework.

Through Active Prototyping, I introduced a novel framework for human-machine collaboration that takes advantage of human dexterity and the ability for thinking through making and the computational capabilities of information processing and storage. Moreover, I took a first step in shaping future technical agendas for more human-centered and creative uses of tools for prototyping and fabrication.
Bibliography


Image Credits

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Figure 5. Image source: Knight and Stiny (2015).

Figure 6. Image source: Mueller, Lopes and Baudisch (2012).

Figure 7. Image source: Follmer, Carr, Lovell and Ishii (2010).

Figure 8: author

Figure 9: author


Figure 11. Image source: Gero (1990).

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Figure 15. Image source: http://history-computer.com/ModernComputer/Software/Sketchpad.html

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Figure 43. Image source: https://www.researchgate.net/figure/Trilateration-technique_fig3_267324735

Figure 44-50. Author.
Appendix
The equations below represent the solution for an absolute-position problem that I used for the calculation of the tool position. One grounded realization of this problem is how to determine the position of a tool tip, given the length of 3 taught-strings used to tie the tip of the tool to 3 separate anchor points that are placed on the periphery of a rotational shaft.

Equations in bold generally indicate more important equations, rather than intermediate derivational steps. Intermediate variables, in addition to “x”, “y”, and “o”, are introduced as needed to simplify the formulation.

Eq 1. \[(x_1 - x)^2 + (y_1 - y)^2 = (d_1 + o)^2\]
Eq 2. \[(x_2 - x)^2 + (y_2 - y)^2 = (d_2 + o)^2\]
Eq 3. \[(x_3 - x)^2 + (y_3 - y)^2 = (d_3 + o)^2\]

Expand Eq. 1,2,3:

Eq 4. \[x_1^2 - 2x_1 x + x^2 + y_1^2 - 2y_1 y + y^2 = d_1^2 + 2d_1 o + o^2\]
Eq 5. \[x_2^2 - 2x_2 x + x^2 + y_2^2 - 2y_2 y + y^2 = d_2^2 + 2d_2 o + o^2\]
Eq 6. \[x_3^2 - 2x_3 x + x^2 + y_3^2 - 2y_3 y + y^2 = d_3^2 + 2d_3 o + o^2\]

Subtract Eq. 5 from Eq. 4:

\[x_1^2 - 2x_1 x + x^2 + y_1^2 - 2y_1 y + y^2 = d_1^2 + 2d_1 o + o^2\]
\[-x_2^2 + 2x_2 x - x^2 - y_2^2 + 2y_2 y - y^2 = -d_2^2 - 2d_2 o - o^2\]

Eq 7. \[x_1^2 - x_2^2 - 2x_1 x + 2x_2 x + y_1^2 - y_2^2 - 2y_1 y + 2y_2 y = d_1^2 - d_2^2 + 2d_1 o - 2d_2 o\]

1. I would like to thank David Wang for his valuable insights on the solution of this problem.
Rearrange Eq. 7:

Eq 8. \[2x_2 - 2x_1 x + 2y_2 y - 2y_1 y + 2d_2 o - 2d_1 o = x_2^2 - x_1^2 + y_2^2 - y_1^2 + d_1^2 - d_2^2\]

Factor:

Eq 9. \[2(x_2 - x_1)x + 2(y_2 - y_1)y + 2(d_2 - d_1)o = x_2^2 - x_1^2 + y_2^2 - y_1^2 + d_1^2 - d_2^2\]

Define new variables, and substitute into Eq. 9.

Eq 10. \[A_1 x + B_1 y + C_1 o = D_1\]
Eq 11. \[A_1 = 2(x_2 - x_1)\]
Eq 12. \[B_1 = 2(y_2 - y_1)\]
Eq 13. \[C_1 = 2(d_2 - d_1)\]
Eq 14. \[D_1 = x_2^2 - x_1^2 + y_2^2 - y_1^2 + d_1^2 - d_2^2\]

Subtract Eq. 6 from Eq. 5 (repeating the same process just completed for Eq. 4 and 5):

Eq 15. \[2(x_3 - x_2)x + 2(y_3 - y_2)y + 2(d_3 - d_2)o = x_3^2 - x_2^2 + y_3^2 - y_2^2 + d_2^2 - d_3^2\]

Define new variables:

Eq 16. \[A_2 x + B_2 y + C_2 o = D_2\]
Eq 17. \[A_2 = 2(x_3 - x_2)\]
Eq 18. \[B_2 = 2(y_3 - y_2)\]
Eq 19. \[C_2 = 2(d_3 - d_2)\]
Eq 20. \[D_2 = x_3^2 - x_2^2 + y_3^2 - y_2^2 + d_2^2 - d_3^2\]
Solve Eq. 10 for x and y:

\[
Eq 21. \quad x = \frac{D_1 - B_1 y - C_1 o}{A_1} \\
\quad y = \frac{D_1 - A_1 x - C_1 o}{B_1}
\]

Solve Eq. 16 for x and y:

\[
Eq 22. \quad x = \frac{D_2 - B_2 y - C_2 o}{A_2} \\
\quad y = \frac{D_2 - A_2 x - C_2 o}{B_2}
\]

Substitute Eq 22. ‘y’ into Eq 21 ‘x’.

\[
x = \frac{D_1 - B_1 \frac{D_2 - A_2 x - C_2 o}{B_2} - C_1 o}{A_1}
\]

\[
Eq 23. \quad x = \frac{D_1 - \frac{B_1 D_2}{B_2} - \frac{B_1 C_2}{B_2} - C_1 o}{\left(\frac{A_1}{B_2} - \frac{B_1 A_2}{B_2}\right)}
\]

Define new variables:

\[
Eq 24. \quad x = E_1 + F_1 o
\]

\[
Eq 25. \quad E_1 = \frac{D_1 - \frac{B_1 D_2}{B_2}}{\left(\frac{A_1}{B_2} - \frac{B_1 A_2}{B_2}\right)}
\]

\[
Eq 26. \quad F_1 = \frac{\frac{B_1 C_2}{B_2} - C_1}{\left(\frac{A_1}{B_2} - \frac{B_1 A_2}{B_2}\right)}
\]
Substitute Eq 22. ‘x’ into Eq 21 ‘y’.

\[ y = \frac{D_1 - A_1 \frac{D_2 - B_2y - C_2o}{A_2} - C_1o}{B_1} \]

Eq 27.

\[ y = \frac{D_1 - A_1 \frac{D_2 + (A_1C_2 - C_2)A_2}{A_2}}{B_1 - A_1B_2} \]

Define new variables:

Eq 28.

\[ y = E_2 + F_2o \]

Eq 29.

\[ E_2 = \frac{D_1 - A_1 \frac{D_2}{A_2}}{B_1 - A_1B_2} \]

Eq 30.

\[ F_2 = \frac{A_1C_2 - C_2}{B_1 - A_1B_2} \]

Substitute Eq. 24 and 28 into Eq. 6, with the goal of setting up a quadratic formula for “o”.

\[ x_3^2 - 2x_3(E_1 + F_1o) + (E_1 + F_1o)^2 + y_3^2 - 2y_3(E_2 + F_2o) + (E_2 + F_2o)^2 \]

\[ = d_3^2 + 2d_3o + o^2 \]

Expand:

\[ x_3^2 - 2x_3E_1 - 2x_3F_1o + E_1^2 + 2E_1F_1o + F_1^2o^2 + y_3^2 - 2y_3E_2 - 2y_3F_2o + E_2^2 \]

\[ + 2E_2F_2o + F_2^2o^2 \]

Rearrange:

\[ x_3^2 + y_3^2 - d_3^2 - 2x_3E_1 - 2y_3E_2 + E_1^2 + E_2^2 - 2x_3F_1o - 2y_3F_2o + 2E_1F_1o + 2E_2F_2o \]

\[ - 2d_3o + F_1^2o^2 + F_2^2o^2 - o^2 = 0 \]

Group terms into a Quadratic Formula:
\[(x_3^2 + y_3^2 - d_3^2 - 2x_3E_1 - 2y_3E_2 + E_1^2 + E_2^2)\]
\[+ (2E_1F_1 + 2E_2F_2 - 2x_3F_1 - 2y_3F_2 - 2d_3) o + (F_1^2 + F_2^2 - 1) o^2 = 0\]

Define variables:

**Eq 31.** \[G o^2 + H o + I = 0\]
**Eq 32.** \[G = (F_1^2 + F_2^2 - 1)\]
**Eq 33.** \[H = (2E_1F_1 + 2E_2F_2 - 2x_3F_1 - 2y_3F_2 - 2d_3)\]
**Eq 34.** \[I = (x_3^2 + y_3^2 - d_3^2 - 2x_3E_1 - 2y_3E_2 + E_1^2 + E_2^2)\]

Use Quadratic Equation to solve for o:

**Eq 35.** \[o = \frac{H \pm \sqrt{H^2 - 4GI}}{2G}\]

The remaining solutions for x and y can be solved for by substituting Eq. 35 Into Eq. 24 and 28.